Lossless Image Compression Using Wavelet and Vector Quantization

ABSTRACT

In this paper, we propose a new low bit rate scheme that combines the Discrete Wavelet Transform (DWT) and Vector Quantization (VQ) for image compression with high Compression Ratio (CR) in order to achieve acceptable quality of output images. Image compression using the combination of DWT with VQ has been considered in many recent works; here in our paper the DWT was performed on several gray scale standard images, each of (256X256) byte. We used 4 images as the training images. Image compression using the combination of DWT with VQ has been considered in many recent works; here in our paper Quantization (VQ) for image compression with high Compression Ratio (CR) in order to achieve acceptable quality of output images are presented.

Our proposed system is showed that the best Wavelet types that you can use for compression is COIF 2. Our proposed system gave better PSNR than JPEG2000 standard at high (CR), above than (51), and this achieved at VQ codebook size (1024 -512) with dimension equal to 256. In the paper a typical system for the application of image compression system and new scientific results are presented.

I. INTRODUCTION

Image compression maps an original image into a bit stream suitable for storage or transmission over suitable channel in a digital medium, such as multimedia communications, integrated services digital networks (ISDN), video conferencing, storage of medical images, archiving of finger prints and transmission of remote sensing images. The number of bits required to represent the coded image should be smaller than that required for the original image, so that one can use less
communication time or storage space Ref. [1]. A fundamental goal of data compression is to reduce the volume of data for transmission or storage while maintaining an acceptable fidelity or image quality. Consequently, pixels must not always be reproduced exactly as the originated also, the Human Visual System (HVS) should not detect the difference between original image and reproduced image. The redundancy (both statistical and subjective) can be removed to achieve compression of the image data. The basic measure for the performance of a compression algorithm is Compression Ratio (CR), defined as a ratio between original data size and compressed data size. In a lossy compression scheme, the image compression algorithm should achieve a tradeoff between compression ratio and image quality.

There are two types of image compression. These are lossless compression and lossy compression. Lossless compression, which is also called noiseless coding, data compaction, entropy coding, or invertible coding, refers to algorithms that allow the original pixel intensities to be perfectly recovered from the compressed representation. Lossy compression algorithms do not allow that Ref. [1]. The compression can be achieved by transforming the data, projecting it on basis of functions, and then encoding the resulted coefficients.

In this paper, we present a new technique of image compression based on wavelet transform and VQ Ref. [2]. Wavelet transform is a recent technique of signal processing Ref. [3] to decompose the original image to one decomposition level. Then, the wavelet coefficients are quantized and encoded using vector quantization Ref. [4]. Multiple Wavelet types with different order and VQ with multiple codebook size and dimension were used as the parameters for compression.

The wavelet coefficients can be quantized, among other methods, by Scalar Quantization (SQ) as in JPEG2000 Standard or VQ as in our work. Comparing SQ versus VQ should take into consideration two aspects; compression ratio achieved and compression quality. VQ gives much higher compression ratio and quality than SQ so the current application uses VQ as will be verified in the result Ref. [5].

So, the remainder of this paper is organized as follows: Section (2) focuses on general background of Discrete Wavelet Transform Decomposition and how to use it in image compression, section (3) focuses on general background of Vector Quantization and how to use it in image compression, section (4) image quality evaluation to quantify the difference, or distortion, between the original image and the reconstructed one, in section (5) we present our algorithm of compression and decompression, and the experimental results are shown in section (6).

II. DISCRETE WAVELET TRANSFORM DECOMPOSITION

DWT can be efficiently used in image coding applications because of its data reduction capabilities. Unlike the case of Discrete Cosine Transform (DCT) which based on cosine functions, basis of DWT can be composed of any function (wavelet) that satisfies requirements of multiresolution analysis Ref. [6]. From that, there exist a very wide choice of functions for basis of DWT. The choice of wavelet depends on contents and resolution of image. DWT has some properties, which make it a better choice for image compression than DCT, especially for image on higher resolutions. The entire image is transformed and compressed as a single data object rather than block by block (as in DCT based system such as JPEG Standard) allowing a uniform distribution of compression error across the entire image. DWT has higher decorrelation and energy compaction efficiency so DWT can provide better image quality on higher compression ratios. Localization of wavelet functions, both in time and frequency, gives DWT potential for good representation of images with fewer coefficients. DWT represents image on different resolution levels.

To effectively transmit digital image, data compression is needed. The original image is transformed so as to remove some of the correlation among pixels; then only the few coefficients with concentrated energies need to be processed Ref. [7]. However, most natural signals and non-stationary
signals need to be analyzed simultaneously in both time and frequency; the well-known Fourier transform (FT) does not give an analysis in time domain. Although the Short Term Fourier Transform (STFT) overcomes the localization in the domain by using a window function, the same sized window must be used for all locations in the time frequency plane. A wavelet transform can be considered to be a generalized STFT; it analyzes the signals while dilating and translating a prototype compactly supported function (wavelet). In other words, wavelet adjusts the window size to fit the signal variance Ref. [8]. Finding the best wavelet basis function to represent an arbitrary signal is still quite complex and takes much time.

Basic principle of DWT firstly is shown in Fig.1., the decomposition of wavelet coefficients doesn’t use a scaling function but based on two filters (L and H) Ref. [6, 9]. LL represents original image to the ½ resolution, corresponding to the low resolution level 1. The three others represent the detail images corresponding to the information visible at the resolution level 1. They are images in the three directions: vertical (HL), horizontal (LH) and diagonal (HH). The union of these four images permits to reconstruct the original image. Only one scale of wavelet decompositions having four different bands are chosen in this research as shown in Fig.1. Starting with original image \( X \) of size 256x256 pixels, the four bands LL, LH, HL, and HH of size 128x128 each for HAAR Wavelet.

The input image \( X \) of size (256 x 256) is first partitioned in the horizontal direction into two halves by the Low pass filter \( H_0(z) \) and the High pass filter \( H_1(z) \). The output of each filter is then down sampled with a factor of 2. Each output is further partitioned in the vertical direction into two halves by the same filters. The output of each filter is again down sampled with a factor of 2, which yields the four bands of the first scale, as shown in Fig.2A. Fig.2B. shows the reconstruction process at the receiver. After the encoded image of each band is decoded and up sampled with a factor of 2, the Low pass image and the High pass image are filtered by the Low pass filter \( F_0(z) \) and the High pass filter \( F_1(z) \), respectively. The output of these two filters is then added together to produce the corresponding reconstructed image \( X \).

The Important properties of wavelet functions in image compression applications are compact support (lead to efficient implementation), symmetry (useful in avoiding de-phase in image processing), orthogonality (allow fast algorithm), regularity, and degree of smoothness (related to filter order or filter length) Ref. [10].
III. VECTOR QUANTIZATION

Vector quantization (VQ) Ref. [11] is one of the attractive techniques for lossy data compression, which can theoretically over perform scalar quantization (SQ). According to Shannon’s rate distortion theory, VQ can always provide a better compression performance than any other coding scheme using SQ Ref. [12].

VQ maps K-dimensional vectors in vector space ($\mathbb{R}^k$) Euclidean space into a finite set of vectors $W = \{W_i; i = 1, 2, ..., N\}$ each vector $W_i$ is called a reproduction vectors (code vectors, reconstruction vectors, codeword). And the set of all the code words is called a codebook. Associated with each codeword, $W_i$, is a nearest neighbor region called Voroni region ($V_i$), and is defined by:

\[ v_i = \{ x \in \mathbb{R}^k : \| x - y_j \| \leq \| x - y_i \|, \text{ for all } j \neq i \} \] (1)

The set of Voroni regions partition the entire space such that:

\[ \bigcup_{i=1}^{N} v_i = \mathbb{R}^k \text{ and } v_i \cap v_j = \phi \text{ if } i \neq j \] (2)

$K$ is the dimension of the vector quantizer and $N$ is the codebook size. One of the parameters of the VQ is the code rate, which measures the bits per vector component. It is used to represent the input vector and gives an indication of achievable accuracy with a vector quantizer if the codebook is well designed. The equation used to calculate this value is defined as:

\[ R = (\log_{2} N) / k \] (3)

This equation measures the number of bits per vector component. In voice waveform coding, $R$ is expressed in bit/sample. In image coding, $R$ is expressed in bits per pixel (BPP).

Hence, the code vector (prototype vector) $W_i$ may be viewed as a pattern -class label (prototype pattern) of the input vectors belonging to the corresponding cell $v_i$. Since VQ performs a mapping of similar input pattern $s$ (input vectors $x$) to similar output patterns (code vectors $W_i$), it may be viewed as a form of pattern recognition where an input pattern is “approximated” by one of a predetermined set (codebook) of standard pattern $s$ (code vectors) Ref. [11].

An important problem of VQ is how to design a codebook that is good for a given source, in the sense that the average difference in representing the input source vectors (input patterns) by the corresponding code vectors (prototype patterns) may be kept to a minimum.

Finally, a vector in VQ can be used to describe almost any type of patterns in an image simply by forming a vector of samples from the images. In practice, the vector size is usually $K=16$ pixels within the image.

A codebook of size $N$ (usually equals 256) is formed by taking a training set of vectors and minimizing the distortion function, which is a measure of the performance of the quantizer Ref. [11]. For images, this represents the bits per pixel (BPP), and therefore specifies the compression ratio for the quantizer. For example, if we set $k=16$ and $N=256$, we get a BPP of $r=0.5$ then (CR=16) by changing $N=128$, we get a BPP of $r=0.4375$ then (CR=18). VQ is typically used to compress images in the spatial domain, but it can also be used to compress transformed coefficients.

Many algorithms have been proposed to design a fast and adequate codebook Ref. [13, 14, 15]. However, they have difficulties in their practical use due to the large amount of computations for codebook design with a lot of input vectors. For example, the Kohonen Competitive Learning (CL) algorithm and its derivative’s algorithms require a lot of iterative computations to obtain an adequate codebook Ref. [4, 16, 17]. The famous algorithm for codebook design is Linde-Buzo-Gray’s Algorithm (LBG) Ref. [18], where codebook is created and optimized using classification. The training set comprised of vectors belonging to different images; it converges iteratively to a locally optimal codebook.

IV. IMAGE EVALUATION

The image quality can be evaluated objectively and subjectively Ref. [10, 19]. Objective methods are based on computable distortion measures. A standard objective measure of image quality is the reconstruction error. Suppose that one has a system in which an input image element blocks $\{x(n), n$
is reproduced as \{y(n)\} \text{ n = 0, 1... N-1}. The reconstruction error \( r(n) \) is defined as the difference between \( x(n) \) and \( y(n) \) and can be expressed as:
\[
r(n) = x(n) - y(n).
\] (4)

Mean-Square-Error (MSE) values over an appropriate sequence length \( M \) and can be expressed as:
\[
MSE = \frac{1}{M} \sum_{i=1}^{N} (x_i - y_i)^2
\] (5)

A standard objective measure of coded image quality is signal-to-noise ratio (SNR), which is defined as the ratio between signal variance \( \sigma_x^2 \) and reconstruction error variance [mean-square error (MSE)] usually expressed in decibels (dB)
\[
SNR(dB) = 10 \log_{10} \left( \frac{\sigma_x^2}{MSE} \right)
\] (6)

When the input signal is an R-bit discrete variable, the variance or energy can be replaced by the maximum input symbol energy \((2^R-1)^2\). For the common case of 8 bits per picture element of input image (e.g., R=8), the peak SNR (PSNR) can be defined as
\[
PSNR(dB) = 10 \log_{10} \left( \frac{255^2}{MSE} \right)
\] (7)

V. THE PROPOSED ENCODING OF WT COEFFICIENTS USING VQ

Encoding and decoding scheme shown in Fig.3 demonstrates the experiments that were conducted by applying our proposed system to image compression based on DWT and VQ. The test images were Lena and Baboon. They are 8-bit gray scale images with a resolution of \( 256 \times 256 \) pixels. In our experiment, we used three types of wavelet families; Haar Wavelet (HAAR), Daubechies Wavelet (DB), and Coiflet Wavelet (COIF). Each wavelet type can be parameterized by an integer that determines the filter order. In our examples, different filter orders are used inside each wavelet type. We have used the following sets of wavelets; DB- N (with N=2, 3, 4, and 5) and COIF – N (with \( N = 1 \) and \( 2 \)). Daubechies and Coiflet wavelets are types of orthogonal wavelets that are compactly supported. Compactly supported wavelets correspond to Finite-Impulse Response (FIR) filters and, thus, they lead to an efficient implementation Ref. [10]. The number of decompositions that we used is equal to one. After decomposing the image and representing it with wavelet coefficients, compression can be performed by quantizing these coefficients using VQ. Totally we used to implement VQ four sets of \((4-16-64-256)\)-dimensional input vectors are generated by dividing these test images into \((2 \times 2)-(4 \times 4)-(8 \times 8)-(16 \times 16)\) pixel blocks respectively.

Our proposed system consists of two main parts (Encoder- Decoder), the input to the encoder is the original image and the output is the image index that is sent through a channel (the channel could be a computer storage, communication channel, and so on). When the decoder receives the index of the previous original image it replaces the index with the associated reconstructed image pixels.

The prerequisite of our system is creating the optimal codebook for VQ. There are several techniques and methods known for creating these codebooks, such as LBG algorithm Ref. [14,18]. The LBG algorithm guarantees that the distortion from one iteration to the next will not increase Ref. [1]. We created our codebooks using four standard images (CMAN- ELAINE- MAN- PEPPERS) as training sequence, which are shown in Fig.4. Also we used other two standard test images (LENA-BABOON) as shown in Fig.5., and Fig.6. Also, as mentioned previously, we used four sets of \((4-16-64-256)\)-dimensional input vectors to VQ.

Encoder and Decoder consisted of the following, Input image 8-bit gray scale images with a resolution of \( 256 \times 256 \) pixels, and Wavelet transform decompose the original image to one level decomposition, then we divide these decomposition level into vector blocks (Block to Vector Conversion) as seen in Fig.3. Then the

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1 In order to evaluate our proposed system, we implemente-d it on a PC consisting of an Intel 850 chip set, RDRAM (256 MB), an Intel Pentium III processor running at 3.00 GHz, and Windows XP Professional platform.
vector block is compared with every codeword in the cookbook (Designed by LBG algorithm) using Euclidean distance formula Ref. [4, 18]. The representative codeword is determined to be the closest in Euclidean distance from the input vector block. The Euclidean distance is defined by:

\[ d(x, w_j) = \sqrt{\sum_{j=1}^{k} (x_j - w_{ij})^2} \] (8)

Once the closest codeword is found, the index of that codeword is sent through a channel. When the decoder receives the index of the codeword, it replaces the index with the associated codeword, and then these codewords are converted to blocks (Vector to Block Conversion) as seen in Fig.3. Then Block Reconstruction collected to reconstruct one level decomposition wavelet coefficient, then Inverse Wavelet transform transformed these coefficients and generated the reconstructed image.

VI. EXPERIMENTAL RESULT

The DWT was performed on the 256x256 byte gray scale images (several standard images). We used 4 images as the training images to train VQ and get the final codebook. These images are standard images (CMAN – ELAINE –MAN - PEPPERS), then we used other two images (LENA - BABOON) to compare our result with the output of these images with JPEG2000 Standard.

The following is our proposed compression result. All papers and works confirmed that, JPEG2000 is the new standard for still image compression. It provides a new framework and an integrated toolbox to better address the increasing needs for compression. It also provides a wide range of functionalities for still image applications, like Internet, color facsimile, printing, scanning, digital photography, remote sensing, mobile applications, medical imagery, digital library and E-commerce Ref. [20, 21, 22, 23 and 24]. So we compare our result from proposed method with JPEG2000 Standard.

In our proposed system, we used many wavelet types such as HAAR, DB2, DB3, DB4, DB5, COIF1, and COIF2 as seen in Fig.3. We used with every wavelet type many of codebook size such as 1024, 512, 256, and 128. Also, for every codebook size we used multiple codebook dimensions such as 4, 16, 64, and 256. We found that the best wavelet type to get best PSNR is COIF2 type as shown in Fig.7. for Lena Image, and Fig.8. for Baboon Image.

From previous step we show that Coif 2 gives the best PSNR at the same Compression ratio, and in the following step COIF 2 results were compared with the result of JPEG2000 Standard. Fig.9. Shows COIF 2 Vs. JPEG2000 Standard for LENA image, and Fig.10. shows COIF 2 Vs. JPEG2000 Standard for BABOON image.

Fig.3. Proposed Compression Model
We can see that JPEG2000 Standard achieved better PSNR than our proposed system at the same CR until (CR= 51) after that our proposed system achieved best PSNR than JPEG2000 Standard i.e. when CR greater than (51), see Table (I). From Table (I) you can see two values (Infinity), first value occurred at (CR=51, codebook size = 1024, and codebook dimensions= 256). Second value occurred at (CR=57, codebook size = 512, and codebook dimensions= 256).

Fig.5. represents (256 X 256) Lena standard original image, Fig.6. represents (256 X 256) Baboon standard original image, Fig.11. represents (256 X 256) Lena image at (CR=51, PSNR= Infinity) for our proposed system, Fig.12. represents (256 X 256) Baboon image at (CR=51, PSNR=Infinity) for our proposed system, infinity here because (MSE=0 and then PSNR= Infinity), Fig.13. represents (256 X 256) Lena image at (CR=51, PSNR=26.558) for JPEG2000 Standard, Fig.14. represents (256 X 256) Baboon image at (CR=51, PSNR=21.699) for JPEG2000 Standard.

VII. CONCLUSIN

In our proposed system, we used many wavelet types such as HAAR, DB2, DB3, DB4, DB5, COIF1, and COIF2. We found that COIF2 wavelet type gave best PSNR. Also, our proposed system gives greater PSNR than JPEG2000 Standard at high compression ration above than (CR= 51). Also, we used codebook with different sizes such as 1024, 512, 256, and 128. Then, for every codebook size, we used multiple codebook dimensions such as 4, 16, 64, and 256. It is found that using (codebook size = 1024, 512 and codebook dimensions= 256) gave best quality i.e. (MSE=0 and then PSNR= Infinity). So, lossless compression using DWT and VQ is achieved.

REFERENCES


Table I  PSNR for COIF 2 Wavelet Vs JPEG2000 for Lena and Baboon Images

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<th>Lena</th>
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Table 1

Fig.4. Codebooks standard images (A) CMAN; (B) ELAINE; (C) MAN; (D) PEPPERS
Fig. 10. COIF 2 Vs. JPEG2000 Standard for Baboon Image

Fig. 11. Represent (256 X256) Lena image at (CR=51, PSNR= Infinity) for our proposed system

Fig. 12. Represent (256 X256) Baboon image at (CR=51, PSNR= Infinity) for our proposed system

Fig. 13. Represent (256 X256) Lena image at (CR=51, PSNR=26.558) for JPEG2000 Standard.

Fig. 14. Represent (256 X256) Baboon image at (CR=51, PSNR=21.699) for JPEG2000 Standard